Research on Basic Quality Classification of Rock Mass Based on Fuzzy Theory

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Abstract. The fuzzy comprehensive evaluation method is widely used in geotechnical engineering. In-depth analysis of the key factors affecting the basic quality of rock mass, according to the basic quality classification of the national standard rock mass, the rock hardness and rock integrity coefficient are used as evaluation indicators. Considering the dynamic change of weights, the subjective weights and objective weights are combined, the normal distribution membership function is modified, and the basic quality fuzzy comprehensive evaluation model of rock mass is constructed. Finally, the results are compared with the national standard grading results in the whole domain. The results show that the coincidence rate of the two methods is 67.2% in the whole domain, and the maximum phase difference is 2 grades; the ratios of the phase difference and the phase difference are 28.5% and 4.3%, respectively.

Introduction

Engineering rock mass grading is a comprehensive evaluation process for the quality and stability of rock mass. It is an important reference for the selection of physical and mechanical parameters of rock mass and is the basic work in rock mechanics and engineering problems [1]. The traditional methods of engineering rock mass classification mainly include RMR method and national standard method. These methods select different evaluation indexes to evaluate rock mass, which are more common in engineering, but each has certain defects: RMR method is applicable in most occasions. However, this classification method is difficult to use when dealing with problems caused by extrusion, expansion, water inrush and its weak rock mass, and the RMR method does not have continuity in scoring, and the single factor score is discrete value, which does not reflect the quality of evaluation factors [2]. Continuous change. The national standard BQ grading method uses qualitative grading and quantitative grading control in engineering rock mass classification, but the situation in which the grading results are inconsistent is not explained. The fuzzy comprehensive evaluation method is based on fuzzy mathematics theory to analyze and evaluate rock mechanics problems [3]. This method has been widely used in predicting rock burst, slope stability analysis and engineering rock mass classification and has made a series of research progress and Results. In this paper, the basic quality classification of rock mass is taken as the research object, combined with the national standard classification characteristics and the current fuzzy theory research results, the corresponding evaluation system is constructed, and the national standard classification results are investigated and studied in the whole domain to make it more perfect.

National Standard Method for Basic Quality Classification of Rock Mass

The national standard uses the BQ value to quantitatively characterize the basic quality of the rock mass. The calculation formula is:

$$BQ = 100 + 3R_c + 250K_v \tag{1}$$

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Where R_c represents the saturated uniaxial compressive strength of rock, unit: MPa; K_v represents the integrity factor of rock mass.

When the R_c of the rock is too large compared to K_v or K_v is too large compared to R_c, the direct substitution into the formula (1) will result in unsafe evaluation of the rock mass. In this regard, the national standard gives two restrictions:

When $R_c > 90K_v + 30$, set:

$$R_c = 90K_v + 30 \tag{2}$$

When $K_v > 0.04R_c + 0.4$, set:

$$K_v = 0.04R_c + 0.4 \tag{3}$$

After obtaining BQ, the basic quality of the rock mass can be graded. The corresponding relationship between the BQ value and the basic quality grade of the rock mass is shown in Table 1.

Table 1 Correspondence between basic mass and BQ of rock mass

Basic quality grade of rock mass	BQ value
I	>550
II	451~550
III	351~450
IV	251~350
V	<251

Grading Principle of Fuzzy Comprehensive Evaluation

First define two sets: set the set $U = \{u_1, u_2, \cdots, u_n\}$, u_i as the factors affecting the quality evaluation of the rock mass, U is called the factor set; $V = \{v_1, v_2, \cdots, v_m\}$, v_j is a possible evaluation level of the rock mass, and V is called a comment set. Make a separate evaluation for each factor u_i in U, determine the degree of membership r_{ij} of the factor u_i to the comment v_j. The evaluation set is:

$$r_i = \{r_{i1}, r_{i2}, r_{i3}, \cdots, r_{im}\} \tag{4}$$

Using n single-factor evaluation sets as the rows of the matrix, the overall evaluation matrix can be obtained:

$$R = [r_{ij}]_{n\times m} = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1m} \\ r_{21} & r_{22} & \cdots & r_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nm} \end{bmatrix} \tag{5}$$

Where $0 \le r_{ij} \le 1$, the essence of which is a fuzzy mapping from the factor set U to the comment set V:

$$f : U \to \tilde{F}(V), u_i \to f(u_i) \tag{6}$$

Where R reflects the fuzzy relationship between factor u_i and comment v_j, which is called fuzzy relational matrix v_j, which is also called membership degree matrix.

In most cases, some factors have a greater impact on the evaluation of the same thing, while some factors have a small impact on the evaluation, that is, the importance of each factor is different. Therefore, it is necessary to define a set of weights for the factors affecting the quality evaluation of rock mass: $W = (w_1, w_2, w_3, \cdots, w_n)$, obviously $\sum_{i=1}^{n} w_i = 1$.

Then the fuzzy comprehensive evaluation can be performed:

$$B = W \circ R = (b_1, b_2, b_3, \cdots, b_m) \tag{7}$$

Among them, $\circ$ stands for generalized fuzzy synthesis operator. B is the hierarchical fuzzy matrix of the comment set V. In the classification of engineering rock mass, the five-level division commonly used in the country is generally adopted, that is, $m=5$. B_j represents the degree of membership of the fuzzy level B of the evaluation level v_j. According to the principle of maximum membership, the level corresponding to max $\{b_j\}$ is comprehensive. The final rating of the judgement.

Grading Process of Fuzzy Comprehensive Evaluation

Selection of Evaluation Indicators and Classification Criteria. Based on the general classification principle of fuzzy comprehensive evaluation and the principle of national standard classification, the rock saturated uniaxial compressive strength R_c and the rock integrity index K_v are used to carry out fuzzy comprehensive evaluation of rock mass basic quality, so that the national standard system and the fuzzy comprehensive evaluation system have the same The basis of the indicator is more intuitive in comparing the effects of the two methods. When using the fuzzy comprehensive evaluation method, the relationship between the factor level of the single factor classification standard and the hardness degree and completeness of the qualitative division according to the national standard is shown in Table 2.

Table 2 Single factor classification criteria

Level	R_c / MPa	K_v
Ⅰ	>60	>0.75
Ⅱ	30~60	0.55~0.75
Ⅲ	12~30	0.35~0.55
Ⅳ	5~15	0.15~0.35
Ⅴ	<5	<0.15

Determination of Single Factor Membership Function. The membership function describes the degree of membership of the fuzzy concept on the domain, and is one of the key steps to determine the fuzzy relationship and carry out fuzzy comprehensive evaluation. For rock mass grading, the most appropriate membership function should belong to the normal distribution membership function:

$$\mu=(x)=e^{-\left(\frac{x-a_0}{c}\right)^2} \tag{8}$$

For a certain interval $[a_1,a_2]$, a_0 is the midpoint of the interval; when one side is the open interval, $a_2=2a_1$.

$$a_0=\frac{a_1+a_2}{2} \tag{9}$$

When $x=a_0$, the membership degree $\mu(a_0)$ is 1; for the endpoint of the interval, that is, the boundary point of the adjacent two phases. When $x=a_1$, $\mu(a_1)=e^{-\left(\frac{a_1-a_0}{c}\right)^2}=e^{\left(\frac{a_1-a_2}{2c}\right)^2}=0.5$, then

$$c=\frac{a_2-a_1}{1.66} \tag{10}$$

The two sides of the interval are treated by the idea of a half-trapezoid and a half-trapped trapezoid. The membership functions of the rock-saturated uniaxial compressive strength R_c are obtained by combining the above analysis with Table 2:

$$\mu_1^{(1)}(x)=\begin{cases} e^{-[(x-90)/36.14]^2}, x<90 \\ 1, x\geq 90 \end{cases} \tag{11}$$

$$\mu_2^{(1)}=e^{-[(x-45)/18.07]^2} \tag{12}$$

$$\mu_3^{(1)} = e^{-[(x-22.5)/9.04]^2} \tag{13}$$

$$\mu_4^{(1)} = e^{-[(x-10)/6.02]^2} \tag{14}$$

$$\mu_5^{(1)}(x) = \begin{cases} e^{-[(x-2.5)/3.012]^2}, x > 2.5 \\ 1, x \le 2.5 \end{cases} \tag{15}$$

Similarly, the membership function of the rock mass integrity coefficient K_v is:

$$\mu_1^{(2)}(x) = \begin{cases} e^{-[(x-0.875)/0.151]^2}, x < 0.875 \\ 1, x \ge 0.875 \end{cases} \tag{16}$$

$$\mu_2^{(2)} = e^{-[(x-0.65)/0.12]^2} \tag{17}$$

$$\mu_3^{(2)} = e^{-[(x-0.45)/0.12]^2} \tag{18}$$

$$\mu_4^{(2)} = e^{-[(x-0.25)/0.12]^2} \tag{19}$$

$$\mu_5^{(2)}(x) = \begin{cases} e^{-[(x-0.075)/0.09]^2}, x > 0.075 \\ 1, x \le 0.075 \end{cases} \tag{20}$$

Determination of the Weight of Evaluation Factors. The weighting method is generally divided into a subjective weighting method and an objective weighting method. The analytic hierarchy process is a kind of subjective weighting method; according to the law of relative importance of factors, the dynamic change of weight is the objective weight. The weights of the two influencing factors are set to w_1 and w_2, respectively, where the subjective weights are $w_1^{(1)}$ and $w_2^{(1)}$, and the objective weights are $w_1^{(2)}$ and $w_2^{(2)}$).

$$w_1 = aw_1^{(1)} + (1-a)w_1^{(2)} \tag{21}$$

$$w_2 = aw_2^{(1)} + (1-a)w_2^{(2)} \tag{22}$$

Where a is a coefficient, $0 < a < 1$, and this paper takes $a = 0.5$.

The sigmoid function is used to describe this constraint. After adjusting the sigmoid function parameters, the following weight functions can be obtained.

$$w_2^{(2)} = \frac{1}{1+e^{-0.04R_c+2.4}} \tag{23}$$

According to $w_1^{(2)} + w_2^{(2)} = 1$, it can be obtained:

$$w_1^{(2)} = 1 - \frac{1}{1+e^{0.04R_c+2.4}} \tag{24}$$

Formula (23) shows that the weight of the rock mass integrity coefficient increases with the increase of R_c, that is, the larger R_c, the more important K_v is in the classification.

Subjective weights are generally constant, and are determined according to the experience of experts, also known as constant weight. When K_v is smaller than 0.1, 0.2, 0.3, and 0.4, it is considered to be slightly more important than R_c; when K_v is moderately 0.5 and 0.6, it is considered to be as important as R_c; when K_v is 0.7, 0.8, 0.9, 1.0, it is considered to be less important than R_c. According to the analytic hierarchy process and the meaning of 1~9 scale, the subjective weights under different K_v values can be obtained through simple calculations.

Table 3 Subjective weights under different K_v

K_v	$w_1^{(1)}$	$w_2^{(1)}$
0.1	0.25	0.75
0.2	0.25	0.75
0.3	0.25	0.75
0.4	0.5	0.75
0.5	0.5	0.5

0.6	0.75	0.5
0.7	0.75	0.25
0.8	0.75	0.25
0.9	0.75	0.25
1	0.75	0.25

Fuzzy Comprehensive Evaluation. After obtaining the weight of the indicator and establishing the membership function of the single factor evaluation, the fuzzy theory can be used to evaluate the basic quality of the rock mass. According to formula (7):

$$B = [w_1 \quad w_2] \circ \begin{bmatrix} \mu_1^{(1)} & \mu_2^{(1)} & \mu_3^{(1)} & \mu_4^{(1)} & \mu_5^{(1)} \\ \mu_1^{(2)} & \mu_2^{(2)} & \mu_3^{(2)} & \mu_4^{(2)} & \mu_5^{(2)} \end{bmatrix} \tag{25}$$

Where $\circ$ is a fuzzy operator. In this paper, we use the weighted average type operator, then the above formula becomes:

$$B = (b_i) = \left(w_1 \mu_i^{(1)} + w_2 \mu_i^{(2)}\right), \quad i = 1,2,3,4,5 \tag{26}$$

According to the maximum membership principle, the level corresponding to $\max\{b_j\}$ is the evaluation level of the basic quality of the rock mass. The flow chart of the rapid fuzzy comprehensive evaluation of the basic quality of rock mass is shown in Figure 2.

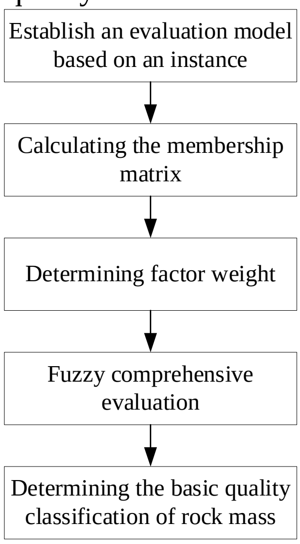

Figure 2 Fuzzy comprehensive evaluation process of rock mass basic quality

Comparative Analysis of Examples

The paper uses discrete points to represent the whole domain, so that k_v takes the values of 0.1, 0.2, ..., 1.0, and R_c takes an integer between 1 and 120, which constitutes a total of 120 pairs. Some of the results of the classification using two methods are shown in Figures 3, 4 and 5.

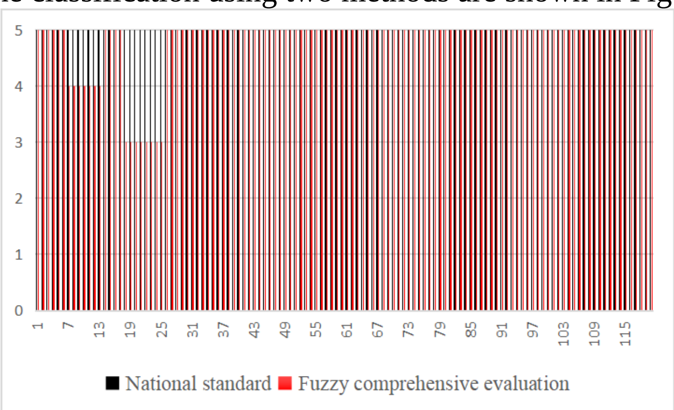

Figure 3 *Kv*=0.1

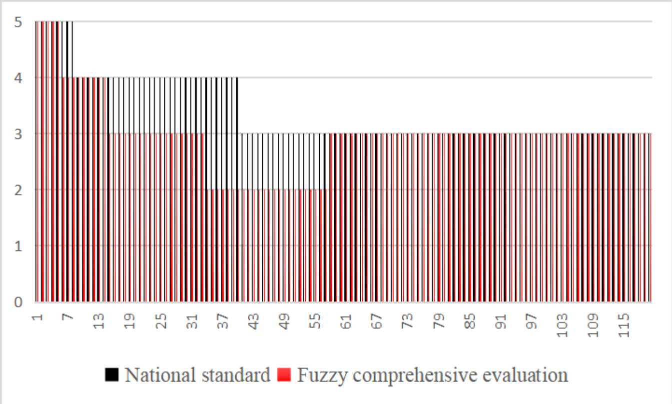

Figure 4 *Kv*=0.5

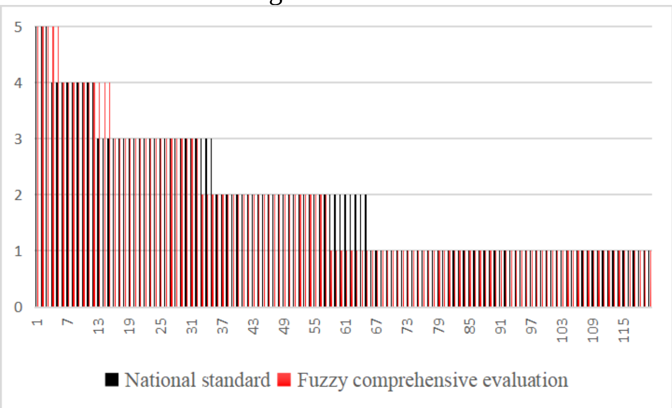

Figure 5 *Kv*=1.0

The coincidence rate of the two methods under different K_v statistics is shown in Table 4.

Table 4 The coincidence rate of national standard classification and fuzzy comprehensive evaluation method under different *Kv*

K_v	Rate of coincidence/%
0.1	85.5
0.2	75
0.3	79.2
0.4	61.7
0.5	60
0.6	55.8
0.7	44.2
0.8	56.7
0.9	67.5
1.0	85.8

It can be seen from Figure 3 that the trend of national standard grading and fuzzy comprehensive evaluation under the same Kv is very close, which indicates that the two methods are in good agreement under these conditions. It can also be seen from the figure that the national standard grading result is always safer than the fuzzy comprehensive evaluation method. It can be seen from Table 4 that when the Kv values are 0.1, 0.2, 0.3, 0.9, and 1.0, the coincidence rates are 85.8%, 75%, 79.2%, 67.5%, and 85.8%, respectively, which indicates that the K_v value is large and small, and is blurred. The comprehensive evaluation method and the national standard classification method have a good agreement; when the K_v value is moderate, the coincidence rate is low, and the anastomosis effect is poor. When K_v is 0.7, the coincidence rate reaches a minimum of only 44.2%. The above comparison results show that the fuzzy comprehensive evaluation method based on dynamic weight has practical value in the study of rock mass basic quality classification, and can be used as an independent research method to classify the basic quality of rock mass.

Conclusion

The fuzzy comprehensive evaluation model of rock mass basic quality is constructed with the hardness of rock and the integrity factor of rock mass as the key factors. The combination of subjective weight and objective weight is used to effectively reflect the change of the relative importance of evaluation indicators with their own values. The process of changing the relative importance of factors has been fully verified. In view of the shortcomings of common methods for verifying with a few examples, 1200 discrete points are used to represent the global scope and verified. Basically, the two methods are compared in the whole domain, which can provide a reference for engineering verification ideas.

Acknowledgements

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